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MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Solitons in Optical Fibres and the Soliton Laser [and Discussion]

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Solitons in optical fibres and the soliton laser

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In this paper, I describe both fundamental and higher-order solitons in optical fibres, their remarkable properties, and the first experimental observation of them. It will be shown that such solitons are easily created and, once formed, are quite stable in the one-dimensional world of single-mode fibres. Consequently, a number of exciting uses have already been found, or have been proposed for them.

One of those uses is in the soliton laser, a mode-locked (short-pulse) laser, whose pulse characteristics are determined by a length of single-mode fibre in its feedback loop. Pulse width scales with the square root of the fibre's length, in accord with $N = 2$ soliton behaviour. The first version of this device, based on a colour-centre laser broadly tunable in the 1.5 μm wavelength region, has already produced pulses as short as 0.13 ps. Compression in a second, external fibre has reduced those pulse widths to less than 50 fs, and reduction by at least another factor of two is considered likely in the near future.

Introduction

In this paper I shall describe solitons in optical fibres and the exciting uses that have been found or suggested for them. Here the solitons are 'envelope' solitons: light pulses whose envelope shapes – in the limit of negligible energy loss – either do not change shape, or else change shape periodically with propagation along the fibre.

For soliton studies one always uses so-called 'single-mode' fibres: fibres that admit of only one possible transverse variation in the light fields. The one-dimensional world of such fibres is an ideal laboratory for the study of solitons, as the twin problems of transverse instability (Hasegawa & Tappert 1973) and of multiple group velocities are eliminated from the outset. Furthermore, the pertinent dispersive and nonlinear properties of such fibres are easily measured and precisely defined.

The dispersive qualities of quartz glass and the loss per unit length of the best single-mode fibres presently available are both shown in figure 1. As will be demonstrated shortly, soliton effects are possible only in the region of 'negative' group velocity dispersion ($\partial v_g/\partial \lambda < 0$). As indicated by figure 1, such dispersion occurs only for wavelengths greater than *ca.* 1.3 μm. (Strictly speaking, the net dispersion of a given fibre is also determined by the ratio of the core diameter to the wavelength. However, such 'modal' contributions can only push the zero of dispersion to longer wavelengths.) Note that the region of negative dispersion includes the region ($\lambda \approx 1.5$ μm) of lowest energy loss (the minimum loss can be as low as 0.16 dB km^{-1}). Thus the 1.5 μm region is nearly ideal for the study of solitons, and all the experiments to be described have been done there.

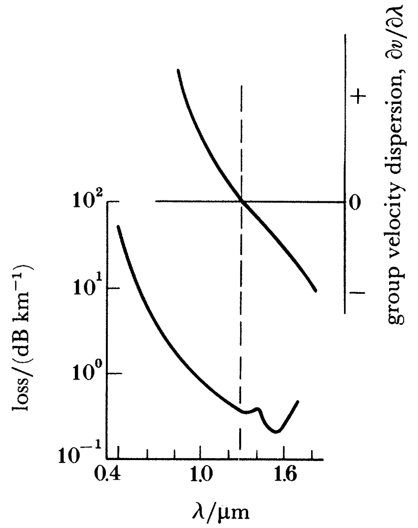

FIGURE 1. Loss of best single mode fibres and group velocity dispersion of quartz glass as functions of wavelength.

Nonlinearity and pulse narrowing

Dispersion alone – regardless of sign – always causes the higher and lower frequency components of a pulse to separate, and thus always serves only to broaden the pulse. Pulse narrowing, and by extension, solitons, are made possible by the fact that the medium is nonlinear, that is, the index of refraction is itself a function of the light intensity:

$$n = n_0 + n_2 I. \tag{1}$$

Here n_2 has the numerical value 3.2×10^{-16} cm^2 W^{-1} and I is the light intensity in compatible units. Although n_2 is rather small, the following points should be kept in mind. (1) Single-mode fibre-core areas, typically *ca.* 10^{-6} cm^2, serve to translate powers of watts into intensities of megawatts per square centimetre. (2) When necessary, the effects of the nonlinearity can be allowed to build up over distances of many metres or even kilometres. (3) For pulse narrowing and soliton production, the required effect is a relatively subtle one, that of 'self phase modulation' (Stolen & Lin 1978).

To understand self-phase modulation, first consider a continuous wave. Such a wave will experience phase retardation (self-phase modulation) in direct proportion to the intensity-induced change in index and to the length of fibre traversed:

$$\Delta\phi = \frac{2\pi}{\lambda} L n_2 I. \tag{2}$$

However, in a pulse such as that shown in figure 2*a*, the rising and falling envelope intensity leads to a similar variation in the degree of phase retardation. The varying phase in turn produces a crowding together and spreading apart of waves in the trailing and leading halves of the pulse, respectively; thus, the frequency 'chirp' shown in figure 2*b* is generated. When such a chirped pulse is acted upon by the fibre's negative group velocity dispersion, the leading half of the pulse, containing the lowered frequencies, will be retarded, while the trailing half, containing the higher frequencies, will be advanced, and the pulse will tend to collapse upon itself, as shown in figure 2*c*. If the peak pulse intensity is high enough (such that the chirp is large enough), the degree of pulse narrowing can be substantial.

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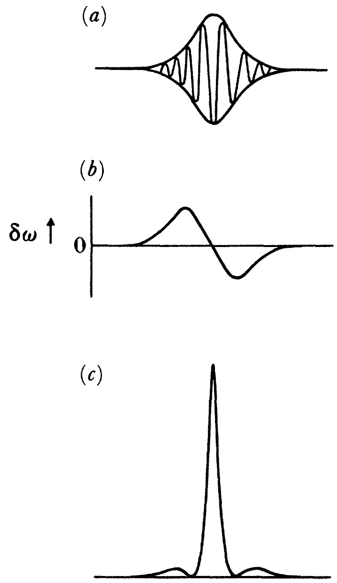

FIGURE 2. (*a*) Optical pulse that has experienced self-phase modulation. (*b*) Corresponding frequency chirp. (*c*) Resultant compressed pulse in a fibre with negative group velocity dispersion (see text.)

The nonlinear Schrödinger equation and solitons

To advance beyond the simple qualitative treatment of pulse narrowing given in the previous section, it is necessary to write down the pertinent differential equation. The development proceeds as follows: first, one assumes that the light pulse can be written as the product of a monochromatic term and a simple envelope function $u(z, t)$, where z is distance along the fibre and t is time. The wave equation for the pulse is then made to yield an equivalent equation for the function u alone. Finally, through a simple linear transformation, the equation in u is reduced to dimensionless form. The result is the nonlinear Schrödinger equation

$$\mathrm{i}\frac{\partial v}{\partial \xi} = \frac{1}{2}\frac{\partial^2 v}{\partial s^2} + |v|^2 v, \tag{3}$$

where v is the dimensionless envelope function, ξ is the dimensionless form of z, and s is the dimensionless version of t.

Despite the great effort required for analytic solution of the nonlinear Schrödinger equation, an analytic technique (the inverse scattering method (Zakharov & Shabat 1973)), and a number of actual solutions are known (see, for example, Satsuma & Yajima 1974). Among these are solutions resulting from an input function of the form

$$v(0, s) = N \operatorname{sech} s, \tag{4}$$

where N is an integer. A few such solutions are shown graphically in figure 3. Corresponding to $N = 1$, one has the fundamental soliton, a pulse that never changes its (sech) shape as it propagates along the fibre. Physically, it represents a condition of exact balance between the pulse-narrowing effect described earlier and the pulse-broadening effect of dispersion alone. (In other words, the pulse shape and amplitude are such that the last two terms of (3) cancel identically, resulting in the equation $\partial v/\partial \xi = 0$.) Thus, for $N < 1$, the purely dispersive effect dominates, and the pulse broadens with propagation, while for $N > 1$, the pulse will always narrow, at least initially. In 'real world' dimensions, the peak *power* corresponding to the fundamental soliton is given by the expression

$$P_1 = 0.776 \frac{\lambda^3}{\pi^2 c n_2} \frac{|D|}{\tau^2} A_{\text{eff}}, \tag{5}$$

where τ is the full width at half maximum (f.w.h.m.) of the input pulse, n_2 has the numerical value given earlier, A_{eff} refers to the fibre core area, and λ and c are the wavelength and speed of light, respectively, both as would be measured in vacuum. The dispersion parameter D reflects the change in pulse delay with change in wavelength, normalized to the fibre length.

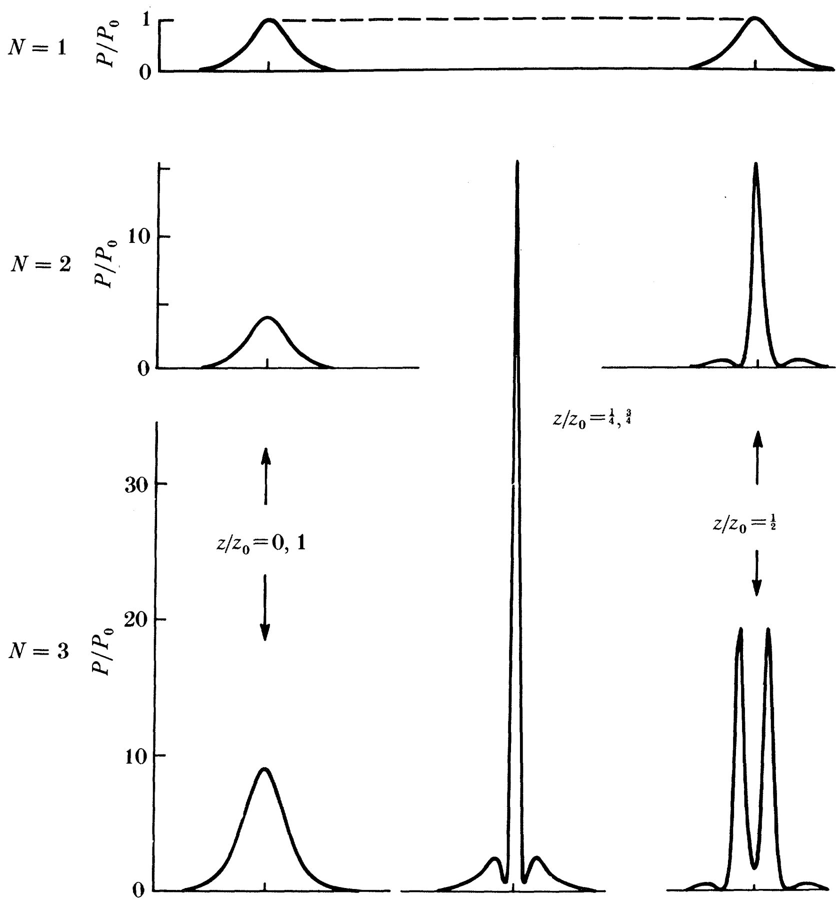

Figure 3. Theoretical behaviour of the fundamental ($N = 1$) and two higher order solitons with propagation (z_0 is one soliton period).

For integers $N \geqslant 2$, sech input pulses always lead to pulse shaping that is periodic with period $\xi = \frac{1}{2}\pi$. In real space, the period is

$$z_0 = 0.322 \frac{\pi^2 c}{\lambda^2} \frac{\tau^2}{|D|}, \tag{6}$$

where the various quantities on the right are defined as before for (5). The peak input powers, of course, are given by the expression

$$P_N = N^2 P_1. \tag{7}$$

For $N = 2$, the behaviour is particularly simple: the pulse alternately narrows and broadens, achieving minimum width at the half period. For greater N, the behaviour becomes more complex, but always consists of a sequence of pulse narrowings and splittings (see figure 3).

It should be noted that although the $N = 1$ soliton shown in figure 3 is unique, the $N = 2$ and $N = 3$ solitons shown there each represent but one member of a continuum. For example, the $N = 2$ soliton can be looked upon as a nonlinear superposition of two fundamental solitons; the continuum of solutions is obtained by varying the relative amplitudes and widths of the two components. (The particular $N = 2$ soliton shown in figure 3 corresponds to components with amplitude and width ratios of 3:1 and 1:3, respectively, and it is the only $N = 2$ soliton to pass through the sech shape at any point in its period.)

Experimental verification

To verify the predicted soliton effects, it is necessary to observe the shapes of pulses as they emerge from a length of fibre. The pulse shapes can be observed by autocorrelation. In that technique, the beam is divided into two roughly equal beams, which, after travelling separate paths, are brought together in a nonlinear crystal; see figure 4. In the arrangement shown there, second harmonic light is generated only if pulses from both beams are simultaneously present in the crystal. Thus the strength of the second harmonic (registered by the photomultiplier) reflects the temporal overlap of the pulses in the two converging beams. A measurement of second harmonic intensity as a function of relative delay then yields the pulse shape in autocorrelation.

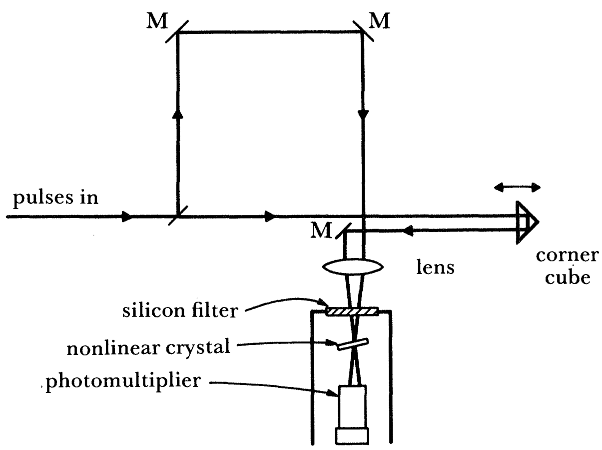

Figure 4. Schematic diagram of apparatus for observing pulse shapes in autocorrelation. Variable delay is accomplished through translation of the corner cube. The silicon filter passes 1.5 μm light but keeps out visible room light. (For principal explanation, see text.)

To explore the full range of soliton effects, one also needs a laser tunable in the 1.5 μm wavelength region and capable of producing a stream of picosecond width pulses of many watts peak power. Recently developed 'mode-locked' colour-centre lasers (Mollenauer & Bloom 1979; Mollenauer *et al.* 1982) are unique in that capability, and have been the pulse source in all the experiments to be described here. It should also be noted that those lasers can be adjusted to yield pulse shapes, as determined both from autocorrelation and by other independent means, that are approximately sech² in intensity. Furthermore, the measured frequency spectrum of the pulses corresponds closely to the Fourier transform of the temporal pulse shape; thus, the pulses contain little or no excess bandwidth. Both characteristics are important to success of the experiments.

As can be seen from figure 3, the most varied changes in pulse shape (with changing input power) are to be observed at the output end of a fibre whose length is one half the soliton period. Figure 5 summarizes the results of an experiment (Mollenauer *et al.* 1980) made with such a half-period fibre. The autocorrelation trace labelled 'laser' describes the colour-centre laser output (fibre input) pulses, and corresponds to a pulsewidth $\tau = 7$ ps (f.w.h.m.) and an approximately sech² shape. The lower row of figure 5 shows the experimentally determined fibre output pulse shapes at certain critical power levels, where one sees, respectively, the expected half-period behaviour of the fundamental and several higher-order solitons.

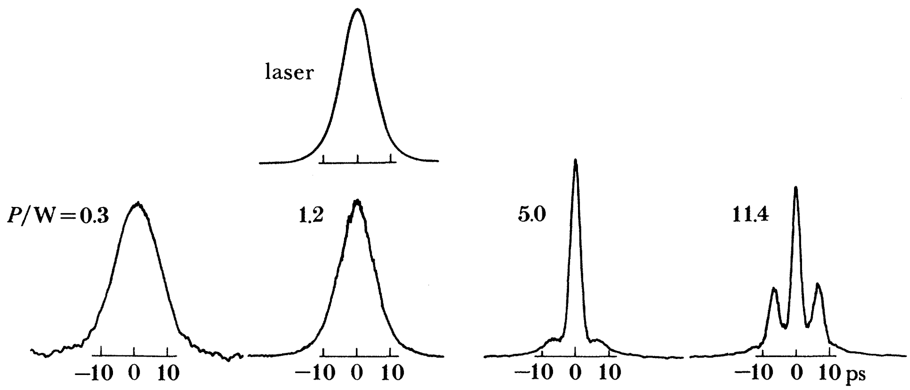

FIGURE 5. Results of experiment with fibre whose length is one-half the soliton period. The top graph, labelled laser, shows the autocorrelation shape of the laser pulses launched into the fibre. The other graphs show autocorrelation shapes of pulses emerging from the fibre for various input powers. $P = 0.3$ W; negligible nonlinear effect; only dispersive broadening is seen. $P = 1.2$ W: return to input pulse width; corresponds to the fundamental soliton. $P = 5$ W: pulse narrowed to minimum width; corresponds to half-period behaviour of the $N = 2$ soliton. $P = 11.4$ W: first well-resolved splitting; corresponds to the $N = 3$ soliton. (Note that the threefold splitting in autocorrelation corresponds to a twofold splitting of the pulse itself.)

It is also satisfying to note that the actual length (700 m) of fibre used in the above experiment agrees rather well with the half-period ($\frac{1}{2}z_0 = 675$ m) calculated from (6) and the appropriate parameters ($\tau = 7$ ps and $|D| = 15$ ps nm^{-1} km^{-1}). The calculated value of P_1 also agrees rather well with experiment. Equation (5) yields $P_1 = 1.0$ W for the parameters just cited and for an effective core area $A_{\text{eff}} \approx 1 \times 10^{-6}$ cm², whereas the average of P/N^2 for the first three solitons yields $P_1 = 1.2$ W.

In a similar experiment (Stolen *et al.* 1983) made with a full-period length of fibre, it has been possible to demonstrate directly the periodicity of the higher-order solitons. In that experiment, at the critical power levels for solitons, both the pulse shapes and the pulse frequency spectra were observed to return to the input values, whereas for intermediate powers, the pulses were narrowed and the frequency spectra correspondingly broadened.

The soliton laser

As may be inferred from the behaviour shown in figure 3, extreme pulse narrowing should be obtained at high soliton number in a judiciously chosen length of fibre. This idea has been refined through theoretical studies and verified by direct experiment; indeed, compression by factors of nearly 30 have been obtained (Mollenauer *et al.* 1983). However, one pays a penalty for such extreme compression: a large fraction of the pulse energy (for $N > 6$, more than half) remains in uncompressed 'wings' surrounding the central spike. The soliton laser (Mollenauer & Stolen 1984*a*, *b*) represents a way to obtain ultrashort pulses without having to pay that penalty.

In the soliton laser, a length of fibre is involved in the laser's feedback loop: pulse compression and solitons in the fibre are used to force *the laser itself* to produce pulses of a predetermined shape and width. This principle has allowed for a previously unknown degree of control in the production of ultrashort pulses. In particular, the soliton laser represents the first and, thus far, only source of femtosecond pulses in the infrared.

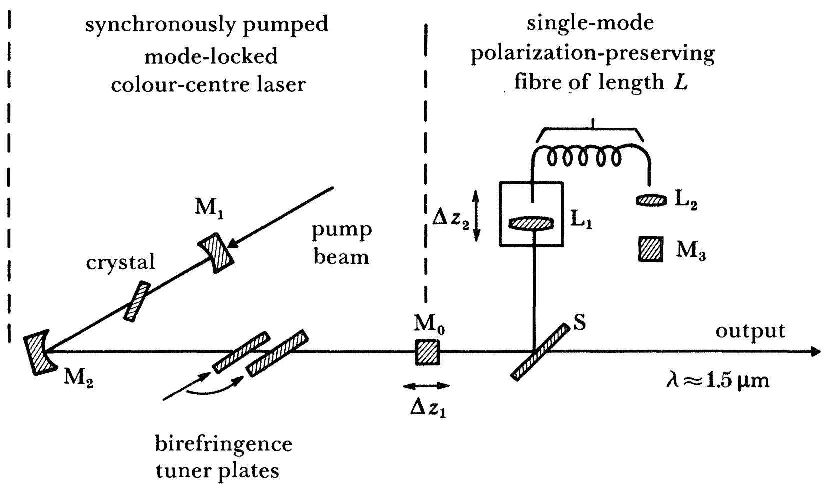

Figure 6. Schematic diagram of the soliton laser. Typical reflectivities: 80% for M_0; 30% for S.

The soliton laser is shown in figure 6. A mode-locked colour-centre laser is coupled through beam splitter S and microscope objective L_1 to a length L of single-mode, polarization-preserving fibre; L_2 and M_3 form an efficient and stable 'cat's eye' retroreflector at the other end of the fibre. The fibre's polarization-preserving ability is vital to successful operation, for otherwise feedback into the polarization-sensitive laser would tend to fluctuate wildly with fibre length, wavelength, and other factors. The input end of the fibre and L_1 are mounted on a common translation stage to facilitate final adjustment (Δz_2) of the optical path length in the fibre arm to be an integral multiple of the main cavity length. (Pulses returned from the fibre must be made coincident with those already present in the main cavity.)

The colour-centre laser and its mode-locking behaviour have been described elsewhere (Mollenauer *et al.* 1982). For present purposes, its significant features are as follows. When pumped with *ca.* 5 W at 1.064 μm, the laser is tunable from *ca.* 1.4 to *ca.* 1.6 μm and produces a stable, non-fading output, up to *ca.* 1 W time-average power at band centre, and finally, by itself, the synchronously pumped laser produces pulses of over 8 ps (f.w.h.m.). (The term 'synchronous pumping' refers to the fact that the pump laser is itself pulsed, with the round-trip time of a pulse in the pumped laser's cavity made exactly equal to the period between pump pulses, through adjustment (Δz_1 in figure 6) of the cavity length.)

With addition of the fibre arm, the device operates as follows. As the laser action builds up from the noise level, the initially broad pulses are considerably narrowed by passage through the fibre. The narrowed pulses, reinjected back into the main cavity, force the laser itself to produce narrower pulses. This process builds upon itself until the pulses in the fibre become $N = 2$ solitons, whose period (z_0) matches the double fibre length ($2L$); at this point the pulses have substantially the same shape after their double passage through the fibre as they had upon entry.

Operation of the laser on $N = 2$ solitons is clearly indicated by the empirically determined dependence of the produced pulse width (τ) on the square root of fibre length (see figure 7), as required for those solitons and the condition $z_0 = 2L$. Also, the values of peak power (P) at the input to the fibre (as inferred from measurement of time-average powers and the pulse width) correspond, to within experimental error, to those values required for $N = 2$ solitons. Note that although for the shortest pulses, the peak powers in the fibre are rather large (nearly 10 kW), the corresponding time-average powers remain modest (under 100 mW), due to the long period (10 ns) between pulses.

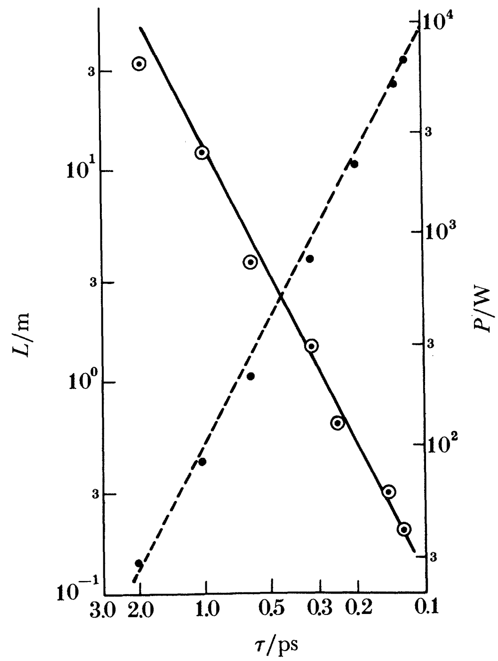

FIGURE 7. Control fibre lengths (encircled points) and peak powers at input to control fibre (solid points) as functions of the obtained laser-output pulse width: ——, $\frac{1}{2}z_0(\tau)$ from equation (6); ----, $P_2(\tau)$ from equations (5) and (7).

It should be noted that the data points of figure 7 represent adjustment (through focusing or defocusing of L_1) of power in the fibre to obtain the narrowest and 'best-shape' pulses. However, corresponding to each fibre length represented in figure 3, there is in fact a finite span of powers for which stable soliton-laser action is obtained and a corresponding range of pulse shapes and widths. For example, figure 8*a* shows a typical 'best-shape' pulse, while the triple-peaked autocorrelation trace of figure 8*b* corresponds to a double-peaked pulse that is produced at higher power levels in the fibre. This variation of pulse shape corresponds to the continuum of $N = 2$ solitons described earlier.

It should also be noted that pulses returned from the fibre are required not only to be coincident with, but also to be in phase with, those circulating in the main cavity. To ensure maintenance of the precise relative cavity lengths (fibre arm and main cavity), the position of M_3 is constantly adjusted by an electronic servosystem. This system has proven quite successful in maintaining stable soliton laser action, even in the face of considerable mirror vibration and drift.

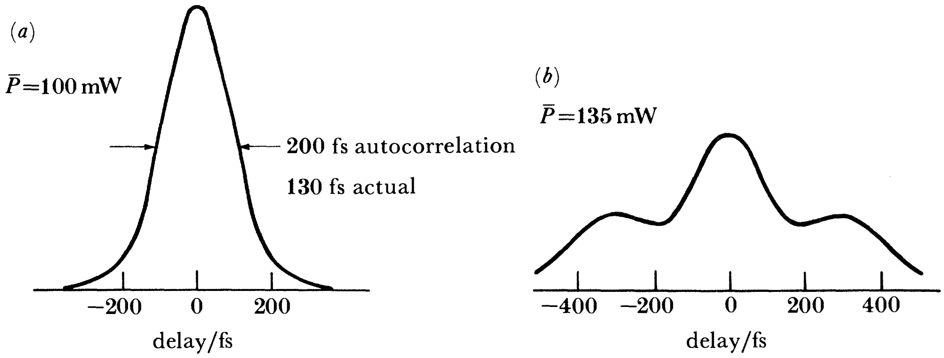

FIGURE 8. Autocorrelation shapes of: (*a*) typical 'best-shape' pulse; (*b*) pulse at higher fibre power. (See text.)

To obtain even shorter pulses than the shortest directly available from the soliton laser, it is possible to use compression in an external fibre. Sufficient power is available to achieve soliton number $N \approx 2$ or 3 in that fibre. The effects of such compression on 150 fs f.w.h.m. soliton laser output pulses are shown in figure 9. The resultant *ca.* 50 fs (f.w.h.m.) pulses have very little pedestal, and it should be possible to remove even that by using the fibre itself as a pulse-height descriminator (Stolen *et al.* 1982). However, the 35 cm compressor-fibre length is longer than optimal (see Mollenauer *et al.* 1983). With optimal fibre length and with perhaps even shorter pulses from the soliton laser, it should be possible to achieve pulse widths in the 20–30 fs range in the very near future. The bandwidths of such pulses (approaching 500 cm^{-1}) will match (or nearly match) the greatest known homogeneous line widths of various semiconductor, colour-centre, or dye transitions. Thus they should allow for the measurement

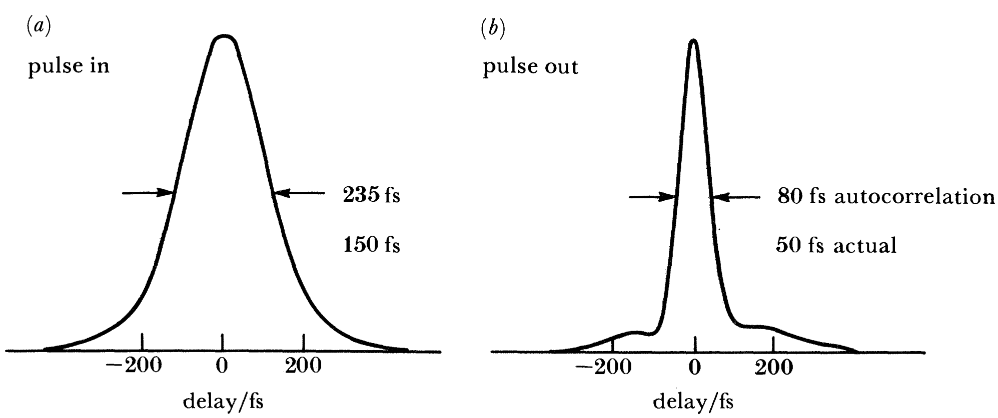

FIGURE 9. Compression of soliton laser pulses in external 35 cm fibre; pulse shapes as seen in autocorrelation.

of relaxation phenomena in those systems to limits of resolution set only by the uncertainty principle itself.

Finally, let it be noted that it should be possible to make a very simple but effective version of the soliton laser from a semi-conductor diode laser, synchronously pumped by electrical pulses and tightly coupled to a length of fibre. Such a device could be a very convenient and inexpensive source of transform-limited pulses of various widths down to perhaps 1 or 2 ps.

Solitons in telecommunications

For the fibre spans encountered in long-distance telecommunications (tens of kilometres in length), loss can no longer be neglected. Nevertheless, it may be possible to create an all-optical, high bit rate, long-distance communications system based on soliton propagation. The proposed scheme (Hasegawa & Kodama 1982) is based on two facts: first, that significant optical gain can be produced in the fibres themselves through the Raman effect and, second, that under the right conditions, when such distributed gain is used to recover the energy of fundamental solitons, their pulse widths are restored as well. Very recent experiments (Mollenauer *et al.* 1985) have confirmed that such recovery of solitons is indeed feasible.

To understand how such soliton recovery works, we need to review a few elementary concepts. The area of a pulse, S, is defined as the integral of the amplitude envelope with time. In terms of the dimensionless amplitude v and dimensionless time s, $S = 1$ for a fundamental soliton. Provided that the loss (gain) *rate* is small enough, both perturbation theory and direct numerical solution of the nonlinear Schrödinger equation show that S will be preserved in the presence of energy loss (gain). Since S scales as the product $A\tau$, where A is the peak pulse amplitude and τ is the pulse width, while the pulse energy E scales as $A^2\tau$, τ itself will scale as E^{-1} as long as the pulse area is preserved. Thus, recovery of the pulse (fundamental soliton) energy through distributed gain will restore the pulse width.

The critical question is, what is the maximum permissible loss rate? Let the energy loss (gain) be described by the equation $\mathrm{d}(\ln E) = -\alpha\, \mathrm{d}z$, where $-\alpha$ is the coefficient of net loss or gain. Direct numerical solution of the nonlinear Schrödinger equation shows that the pulse area is very well preserved, and hence the soliton recovery works well, as long as the product αz_0 is less than *ca.* 0.05.

In the experiments, 10 ps (f.w.h.m.) pulses from a mode-locked colour-centre laser operating at $\lambda = 1.56$ μm were made to propagate in one direction along a 10 km length of fibre, while light from a continuous-wave colour-centre laser operating at $\lambda = 1.46$ μm was introduced from the opposite end. The measured loss figures for the fibre at the signal (pulse) and pump wavelengths were 0.18 and 0.29 dB km^{-1}, respectively. With the pulse input power carefully adjusted to the value required for fundamental solitons, and the pump power adjusted such that the overall fibre loss was exactly compensated by Raman gain, autocorrelation traces of the fibre input and output pulses were virtually indistinguishable. Without the Raman gain, the fibre output pulses were *ca.* 1.5 times broader than at the input, a result again in close accord with theoretical prediction. (The overall fibre loss of 1.8 dB corresponds to energy reduction by a factor of *ca.* 1/1.5.) Sufficient pump power is available from our colour-centre laser to allow for the compensation of loss in fibre spans of 50 km or more. (The required pump power for the present experiment was approximately equal to 125 mW.)

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Recent computer simulations (Hasagawa (1984) and work by Mollenauer and colleagues, which is in preparation for publication) have shown that solitons can be made to propagate stably for thousands of kilometres in periodically gain-compensated fibres. Furthermore, they show that stable transmission is possible for 'large' z_0 (z_0 comparable to the amplification period), in addition to the régime of 'short' z_0 described above. Although experimental demonstration has not yet been attempted, there is now good reason to believe that such a periodic-amplification scheme can be made to work. If so, then the tremendous, but so far merely hypothetical, information-carrying capacity of optical fibres will finally become available in practice.

Relation of this work to theoretical studies

In conclusion, I would like to indicate briefly the stimulating effect this work has already had, or is expected to have, on theoretical studies. In the first place, at several times in this work, there has been a need for the assured precision that can come, where large N is involved, only from *analytic* solutions of the nonlinear Schrödinger equation. In response, Gordon (1983) has found a way to reduce the $2N$ simultaneous algebraic equations involved in the inverse scattering method to just N such equations; this has allowed, for the first time, practical realization of analytic solutions for $N > 3$. (With computer inversion of the matrices, solutions are now available for N as great as *ca.* 20; see Mollenauer *et al.* 1983) Gordon (1983) has also derived the equations of motion for two nearly degenerate solitons (energies and velocities almost equal); this study of copropagating solitons and the attractive forces between them has significant implications for the use of solitons in telecommunications. Additionally, H. A. Haus and M. N. Islam of M.I.T. have recently generated a detailed theory (manuscript in preparation) of the soliton laser; in so doing they have been forced to explore the effects of various perturbations on $N = 2$ solitons and to examine the stability of the resultant solutions. Others are beginning to study soliton propagation in the presence of fibre birefringence. (Unavoidable strains make 'single mode' fibres slightly birefringent, and hence potentially bimodal. The question is, when does such birefringence begin to affect soliton propagation?)

Many new questions, as yet unanswered, have been raised in connection with the femtosecond pulses generated by the soliton laser. For example, such pulses violate the assumption, inherent in the Schrödinger equation, that dispersion is a constant over the bandwidth of the pulse. How should such variable dispersion affect soliton propagation? It is also tacitly assumed in the nonlinear Schrödinger equation that the nonlinearity is instantaneous, yet that assumption must break down on some sufficiently short timescale. Is the response time long enough to yield sensible effects on the propagation of femtosecond pulses, and if so, what are they? Finally, will there be noticeable effects on soliton propagation when the pulse width becomes shorter than the propagation time across the fibre core, as occurs for pulse widths of just a few tens of femtoseconds? One hopes that the challenge to the theorist posed by these questions will be answered, now that the possibility of corresponding experiment has been made real.

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Discussion

R. K. Bullough (*UMIST, Manchester, U.K.*). I have one comment and what is essentially one question. The comment is that it is very nice to see this beautiful physical example of 'true' soliton behaviour: that is, with the perfect (or almost perfect?) soliton collision property. Eight or nine years ago we tried to persuade Hyatt Gibbs, also of Bell Laboratories (but Murray Hill!), to demonstrate soliton collisions in self-induced transparency. He had already demonstrated soliton break-up in ^{87}Rb vapour in 1972, and with B. Bölger (Philips, Eindhoven) he observed perfect break-up into solitons of transform-limited pulses in Na vapour in 1976. However, the self-induced transparency equations are integrable for pulses travelling in one space direction only, so the solitons have to overtake each other to collide as solitons. Even Gibbs found this experiment much less easy than colliding oppositely directed pulses. In *this* case of self-induced transparency the pulses do not collide as perfect solitons: there is an interaction that Gibbs & Bölger (1977) were able to observe.

We were subsequently able to simulate these interacting pulse collisions by using a much improved version of Whitham's averaged Lagrangian technique (cf. Bullough *et al.* 1979; Jack 1978) (cf. my remarks on the paper by Professor Keller (this symposium)).

My 'question' arises out of these remarks: how perfect are these pulse collisions in the soliton laser, and how perfect are the individual pulses? Are they exact (or exact enough) hyperbolic secant envelopes? Do they correspond quantitatively to the input data? (How precisely can that input data be specified anyway: to 5% or to something very much better?) Although the $N = 2$ soliton oscillation is beautifully observed, is it quantitative? Really my question is: how accurately does the nonlinear Schrödinger equation, as such, already describe in quantitative terms what is actually observed? In particular, I think Dr Mollenauer showed a single soliton-like pulse shape with a pronounced 'platform' underneath it. What is the origin of this 'platform'? I know Dr Mollenauer has already mentioned a number of possible corrections: effects of strain birefringence, variable dispersion, response time of the nonlinearity. My question is, therefore, how good in quantitative terms is the nonlinear Schrödinger equation now, and what are the important corrections to make to it? What is the origin of the platform mentioned?

These are really beautiful experiments.

References

Bullough, R. K., Jack, P. M., Kitchenside, P. W. & Saunders, R. 1979 *Physica Scr.* **20**, 364–381.
Gibbs, H. M. & Bölger, B. 1977 In *Coherence and quantum optics*, vol. IV (ed. L. Mandel & E. Wolf), pp. 759–765. New York: Plenum.
Jack, P. M. 1978 Ph.D. thesis. University of Manchester.

L. F. Mollenauer. Except for those inaccuracies, already mentioned, that may arise with extremely short ('femtosecond') pulses, the nonlinear Schrödinger equation is thought to be nearly exact in its description of minimum-bandwidth pulses in single-mode fibres. Therefore, the experiments were not conceived as a rigorous test of that equation. Rather, our object was to demonstrate that pulses, conforming to some of the simpler soliton solutions of that equation, could easily be generated, observed, and made use of in fibres. By so doing, we hoped to make 'real' and accessible that which, until then, had been considered merely hypothetical, and whose potential in the practical world had been viewed with a certain amount of scepticism. However, I hope I have not given the impression that agreement between theory and the experiments described here is primarily qualitative; that is simply not true. For example, consider our data (figure 7) to characterize the soliton laser: the extensive quantitative agreement shown in this figure is about as close as could possibly be hoped for. Even the results of our very first experiment (figure 5) show close agreement between theoretical prediction and the measured pulse powers, widths, shapes and the soliton (half) period. Finally, to answer Professor Bullough's question about the pulse shape with a 'platform' underneath it: that pulse was shown to illustrate the extreme compression that can occur at high soliton number in a fibre of the appropriate length, and the platform represents the essentially uncompressed, low-intensity wings of the original pulse. Such wings, containing half or more of the pulse energy, are predicted by solution of the nonlinear Schrödinger equation for large N (Mollenauer *et al.* 1983). To sum up, a well behaved source of minimum bandwidth pulses such as our mode-locked, colour-centre laser, and with the well characterized, uniform fibres presently available, one can indeed simulate predicted soliton effects with considerable accuracy. The very nature of solitons assist in that aim: if the launched pulse is at first not an exact soliton, it will soon form into one, together with a non-soliton residue that is lost through dispersion.

N. C. Freeman (*School of Mathematics, The University of Newcastle upon Tyne, U.K.*). The $2N\times 2N$ determinant of the nonlinear Schrödinger system to an $N\times N$ determinant referred to by Dr Mollenauer is equivalent to a similar reduction for the Davey–Stewartson equation (Freeman 1984) when the y-coordinate was eliminated.

Reference

Freeman, N. C. 1984 *I.M.A.Jl appl. Phys.* **32**, 125–145 (Appendix B).

T. Beth (*Department of Computer Science, Royal Holloway College, University of London, U.K.*). In which distances would the Raman-effect Laser pumps have to be distributed over an optical fibre transporting solitons? In my view it seems to be necessary to encode messages properly if the energy pumped in by a Raman gain pump shall suffice. Has Dr Mollenauer already studied such coding procedures?

L. F. Mollenauer. With bidirectional pumping in the best fibres available, Raman-effect laser pumps spaced about 50 km apart would result in signal-pulse energy variation along the fibre of no more than approximately $\pm 15\,\%$. Computer simulation has shown that fundamental soliton pulses can propagate stably for thousands of kilometres along such a periodically pumped fibre, provided that the soliton period is greater than about a quarter of the laser pump spacing. The signals would be digitally encoded in the usual way, that is, the presence of a pulse in a given time slot would represent a 'one', while its absence there would represent a 'zero', or vice versa. In a special, low-dispersion fibre, the soliton signal pulses would have peak powers of the order of 10 to 30 mW, would each be of the order of 10 ps wide, and have a minimum separation between adjacent pulses of about 100 ps. So, the time-average power in the signal pulse stream would be at most one or two milliwatts, and would produce negligible depletion of the pump power. The fundamental bit rate would be of the order of 10 GHz, and with wavelength multiplexing, the overall bit rate for a single fibre could be well in excess of 100 GHz. Such a rate would be two orders of magnitude greater than the best presently achieved with conventional technology!

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